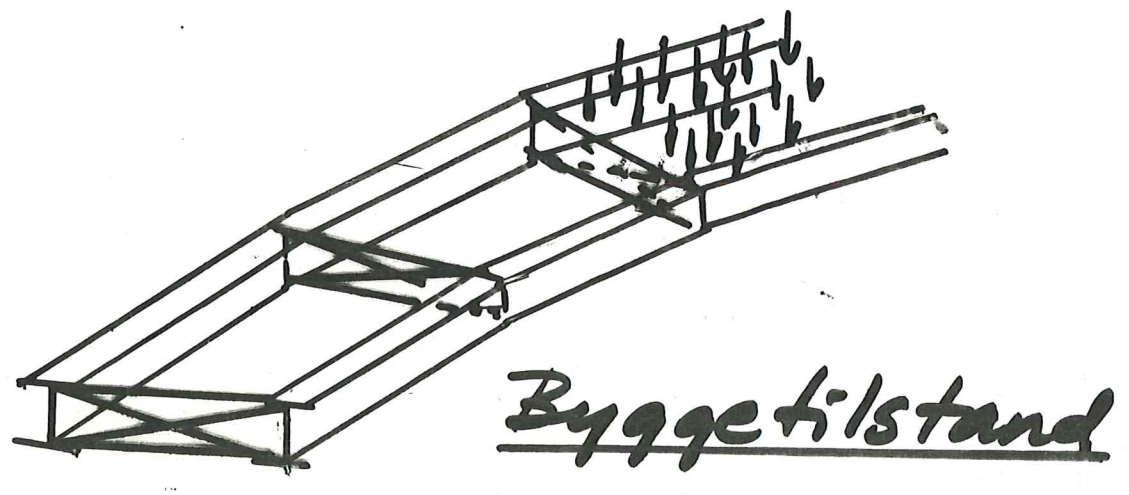
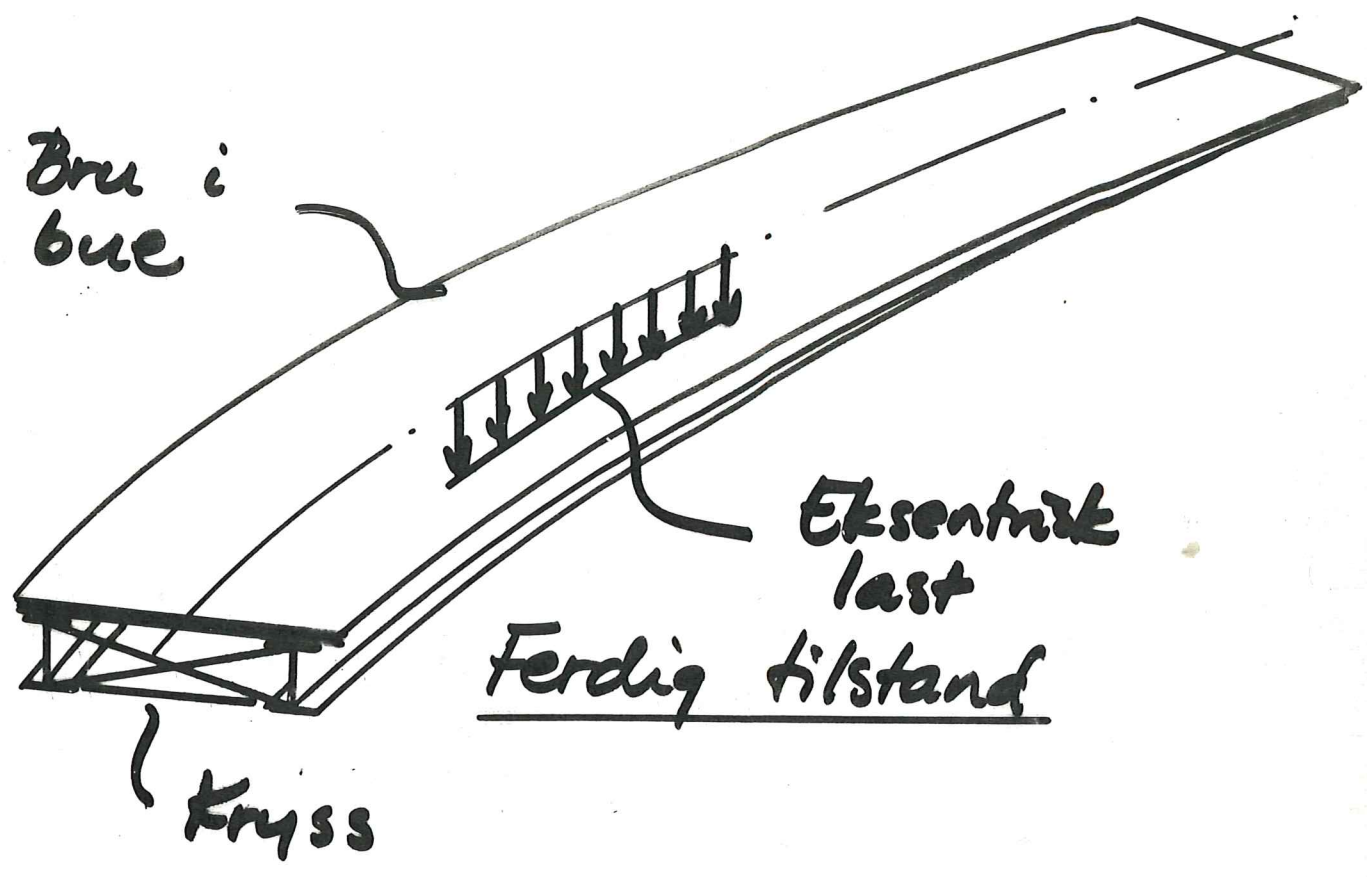
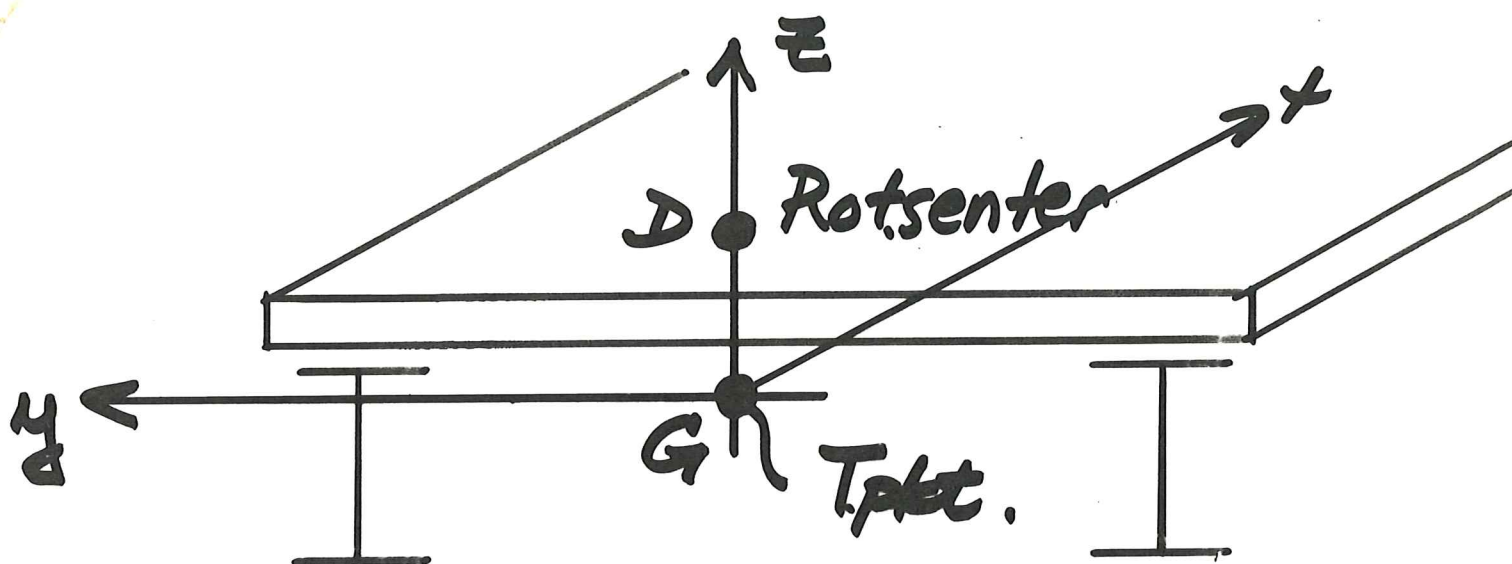


SAMVIRKE - BRU - PROGRAM



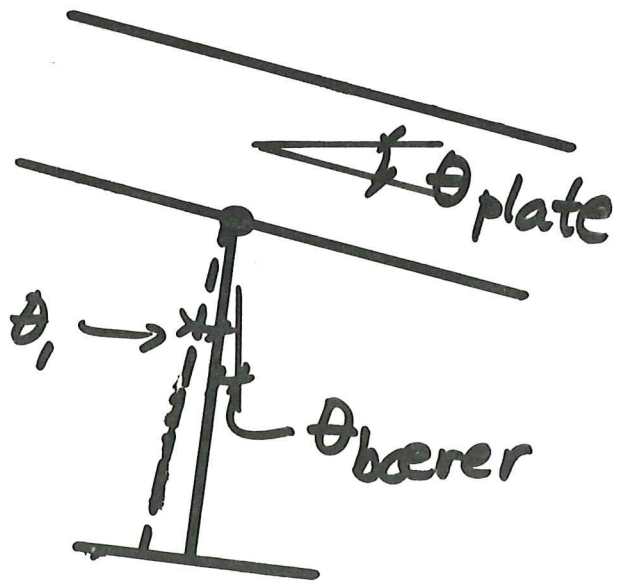
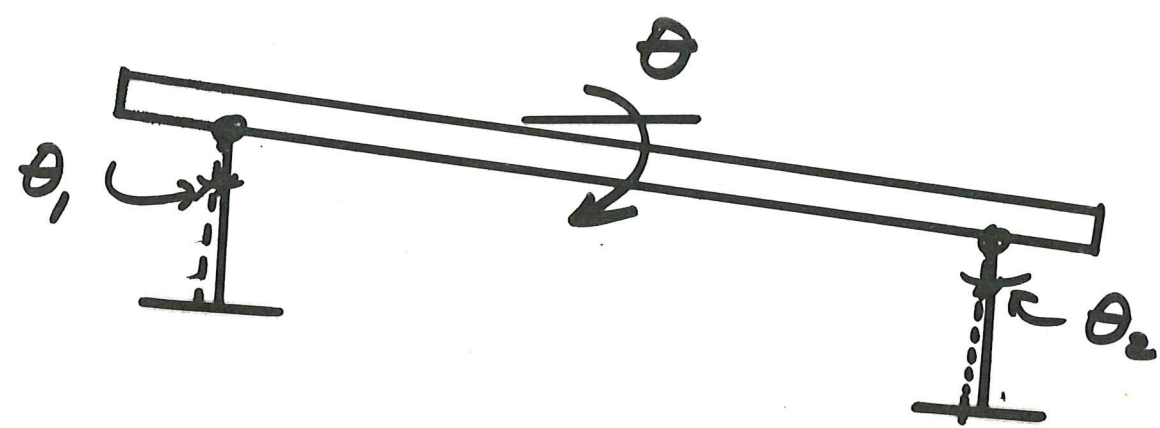


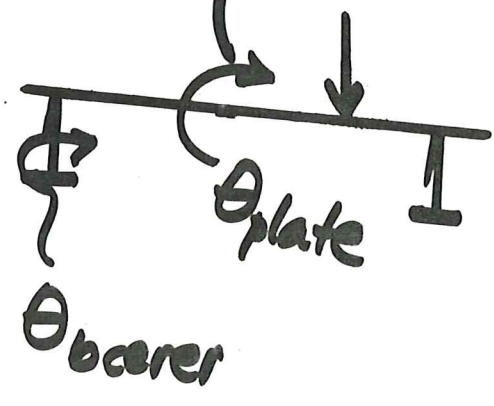
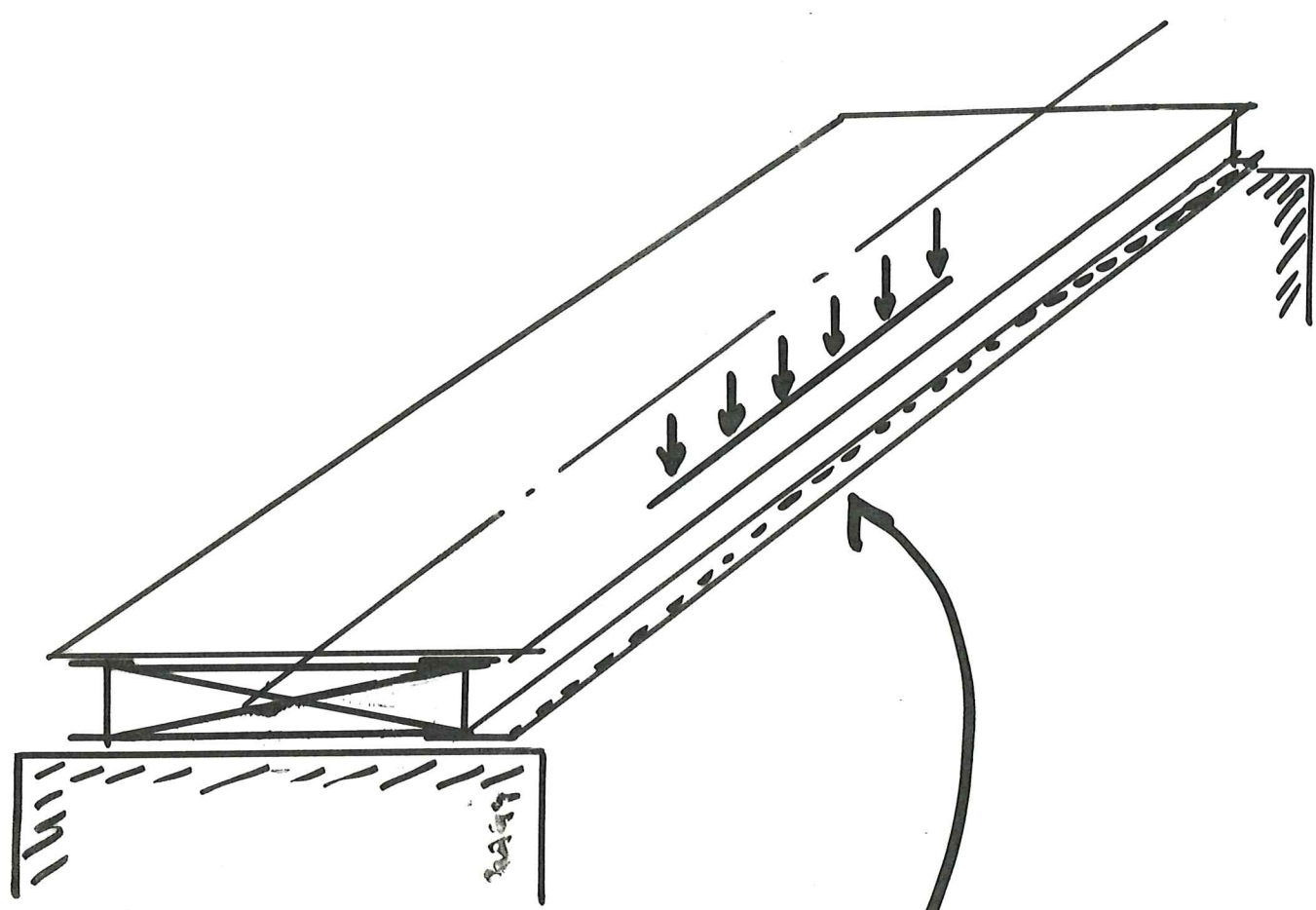
$x =$ langs brua.

$y, z =$ hovedakser g.j. t.pkt.

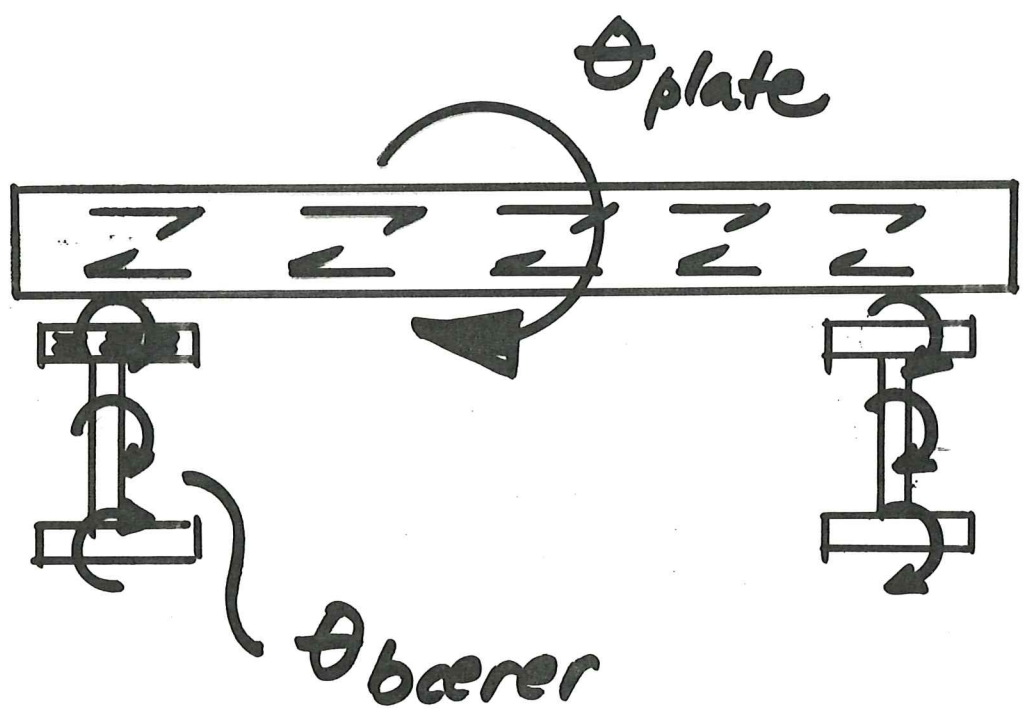
$$\int y \, dA = \int z \, dA = 0$$

$$\int yz \, dA = 0$$





St. Venant - torsjon



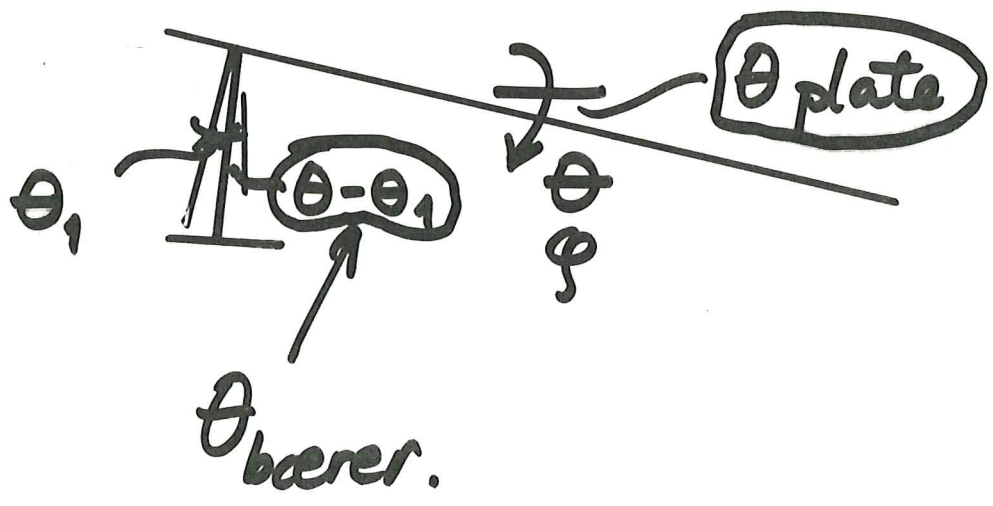
$$P_{ix} = \frac{M_x}{GI_t}$$

$$GI_t = \frac{1}{3} \sum G_i b_i t_i^3$$

Ma modifieres!

$$M_x = (GI_t)_{plate} \cdot \theta_{plate}$$

$$+ \sum_{b\ddot{a}erere} GI_t \cdot \theta_{b\ddot{a}erer}$$



Hvelvning

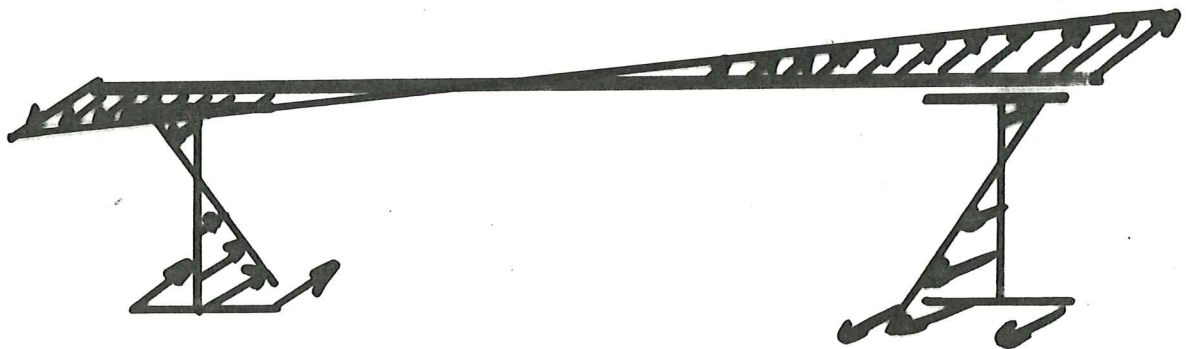
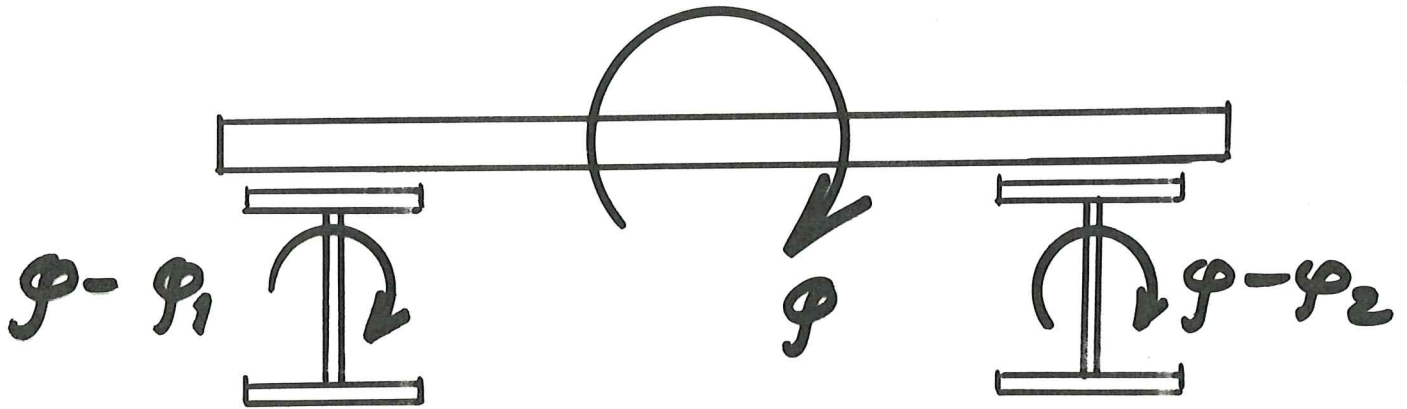
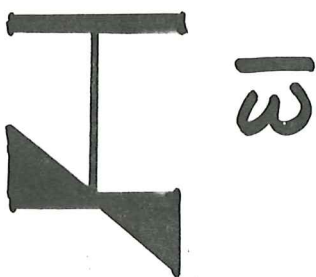


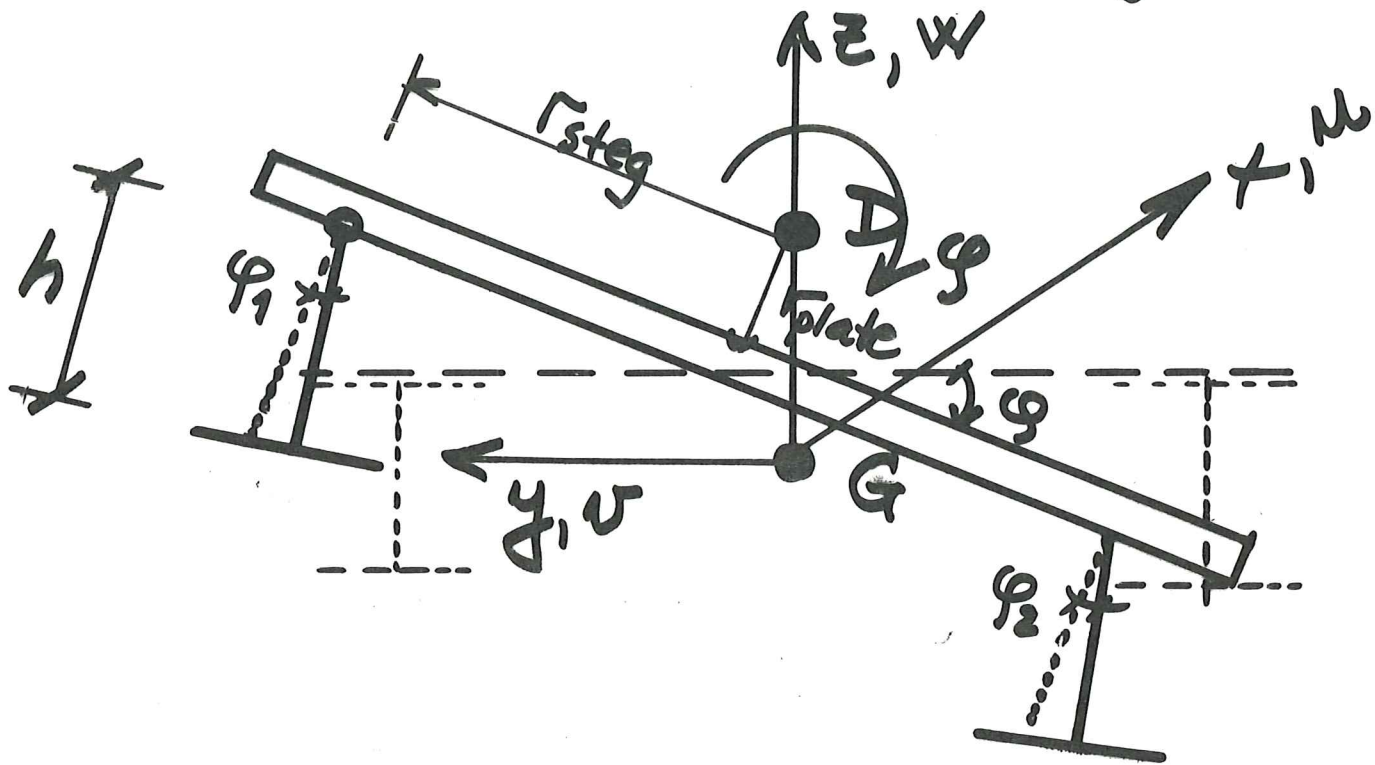
Plate: $\sigma_x = -E\omega \varphi_{,xx}$

Bjelke: $\sigma_x = -E\omega \varphi_{,xx} + E\bar{\omega} \varphi_{,xx}$



Bare bidrag
under flens

Ligninger for hvelvning

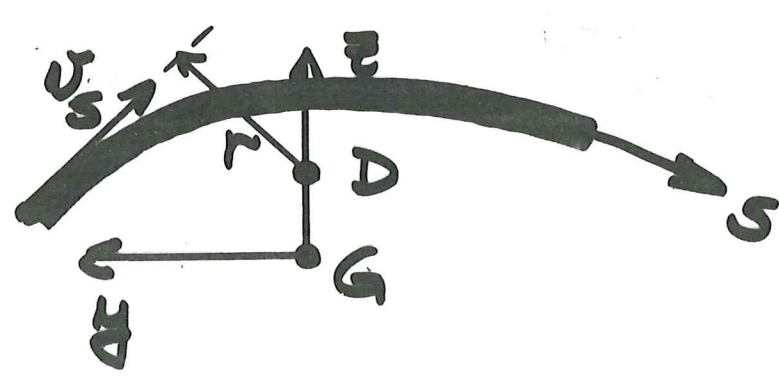


Forstyvning langs flate :

$$\underline{v_s = \varphi \cdot r}$$

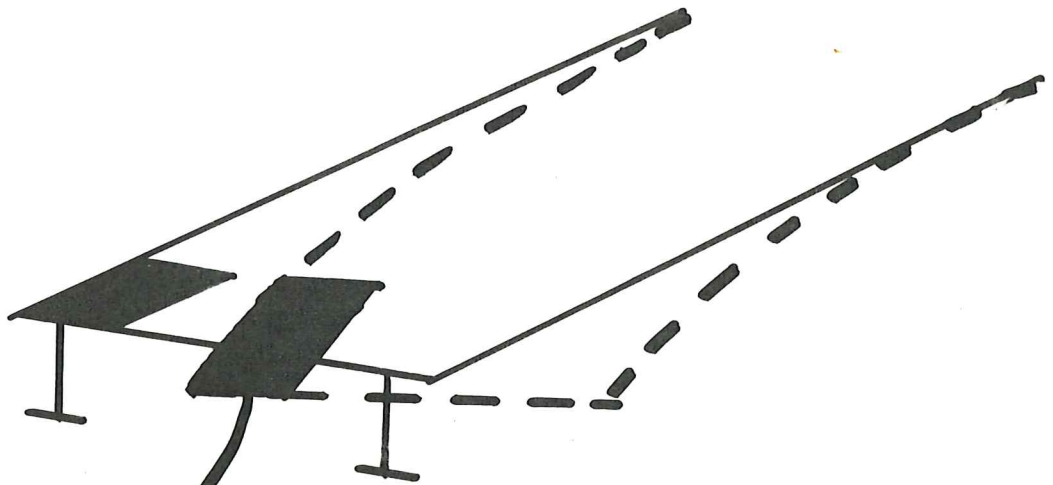
Korreksjon for underflens :

$$\underline{\bar{v}_s = \varphi \cdot r - \varphi_1 \cdot h}$$

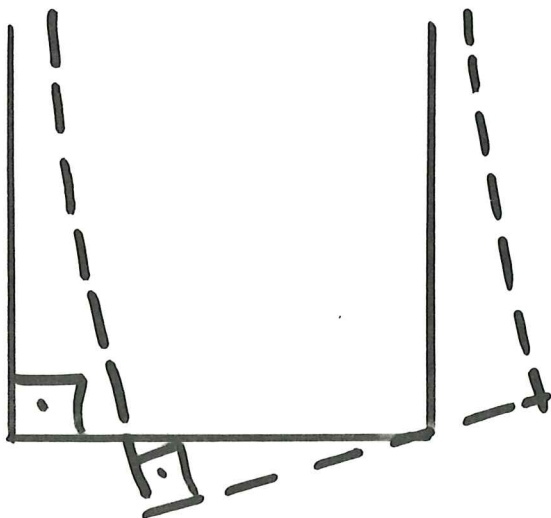


Vlasov teori :

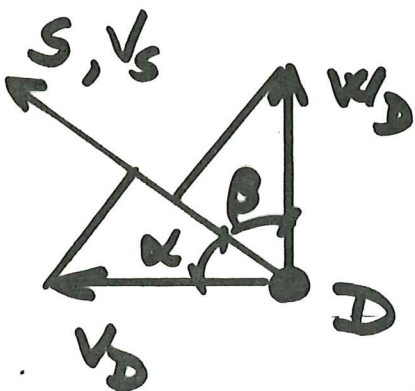
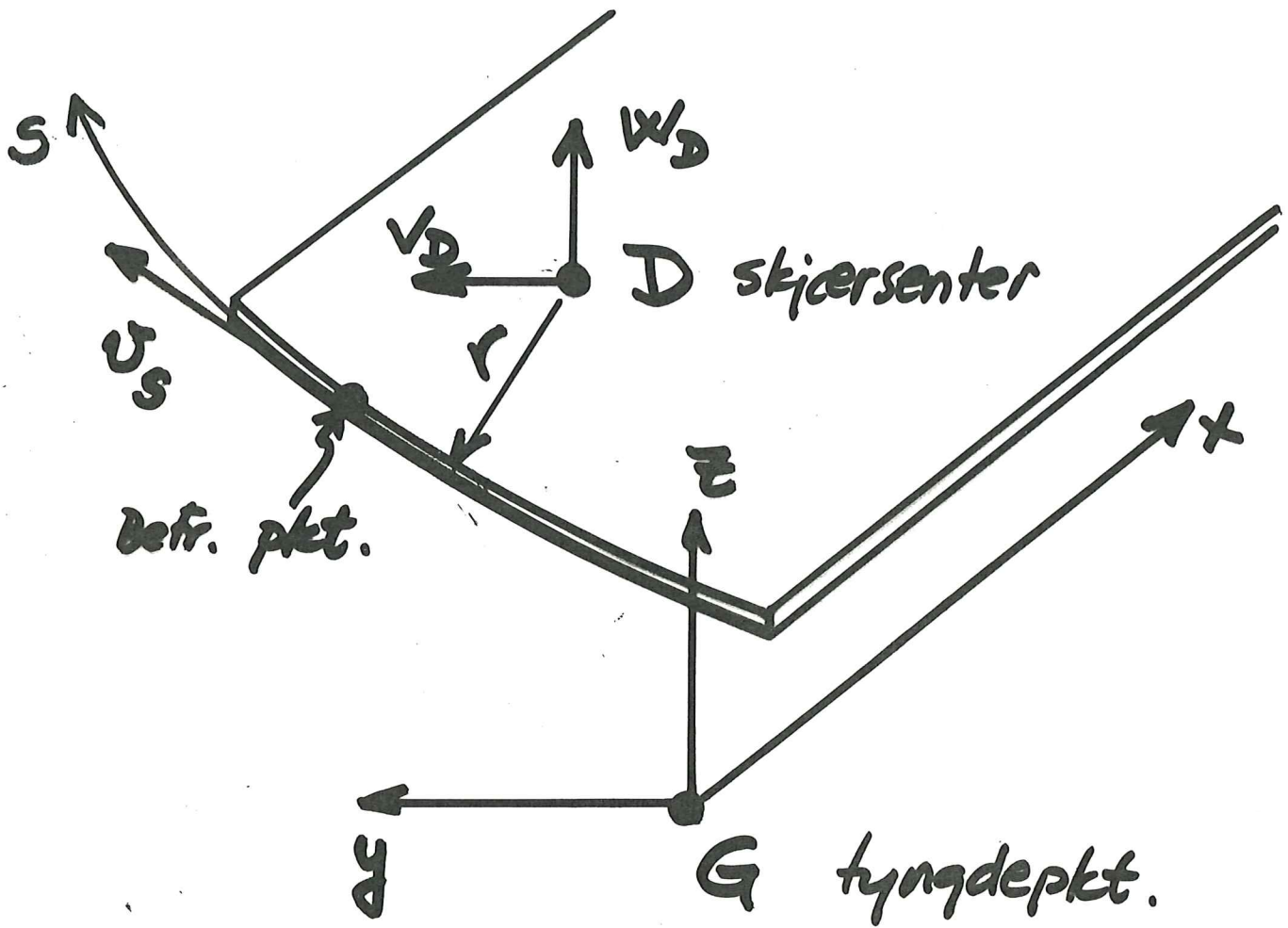
$$\sigma_{xs} = \frac{\partial u}{\partial s} + \frac{\partial v_s}{\partial x} = 0$$



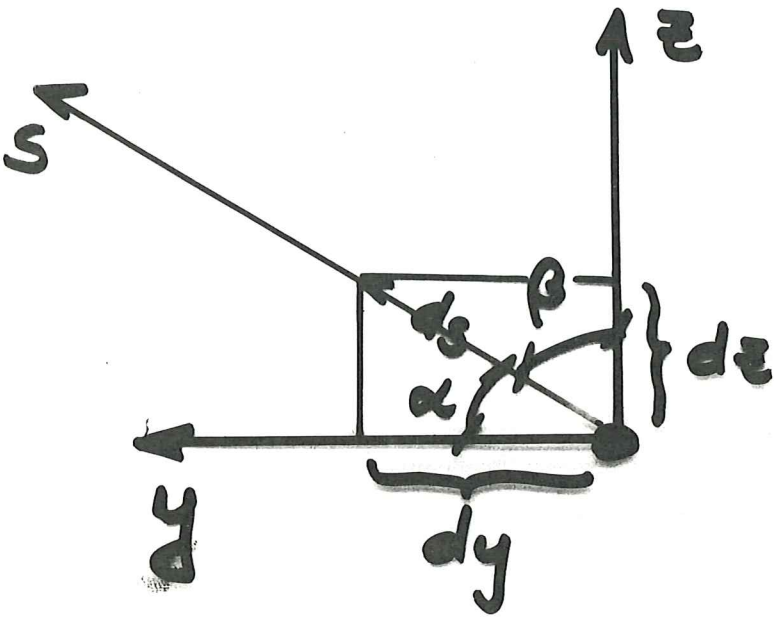
Ingen skjærendeformasjon
Bjelke uten skjær



Komplett forskyvning:



$$v_s = \varphi \cdot r + v_D \cdot \cos \alpha + w_D \cdot \cos \beta$$



$$\cos \alpha = \frac{\partial y}{\partial s} ; \quad \cos \beta = \frac{\partial z}{\partial s}$$

$$V_s = \rho \cdot r + v_D \cdot \frac{\partial y}{\partial s} + w_D \cdot \frac{\partial z}{\partial s}$$

Vlasov:

$$\frac{\partial u}{\partial s} = - \frac{\partial V_s}{\partial x}$$

$$= - \frac{\partial \rho}{\partial x} \cdot r - \frac{\partial v_D}{\partial x} \cdot \frac{\partial y}{\partial s} - \frac{\partial w_D}{\partial x} \cdot \frac{\partial z}{\partial s}$$

Konst. traversn. over element.

Aksialforskyvning:

$$u(x, s) = u_0(x) - \varphi_{,x} \cdot \int_s r ds - v_{D,x} \cdot y - w_{D,x} \cdot z$$

$u_0(x) = \text{int. konstant}$

Innfor hvelvningsparameteren

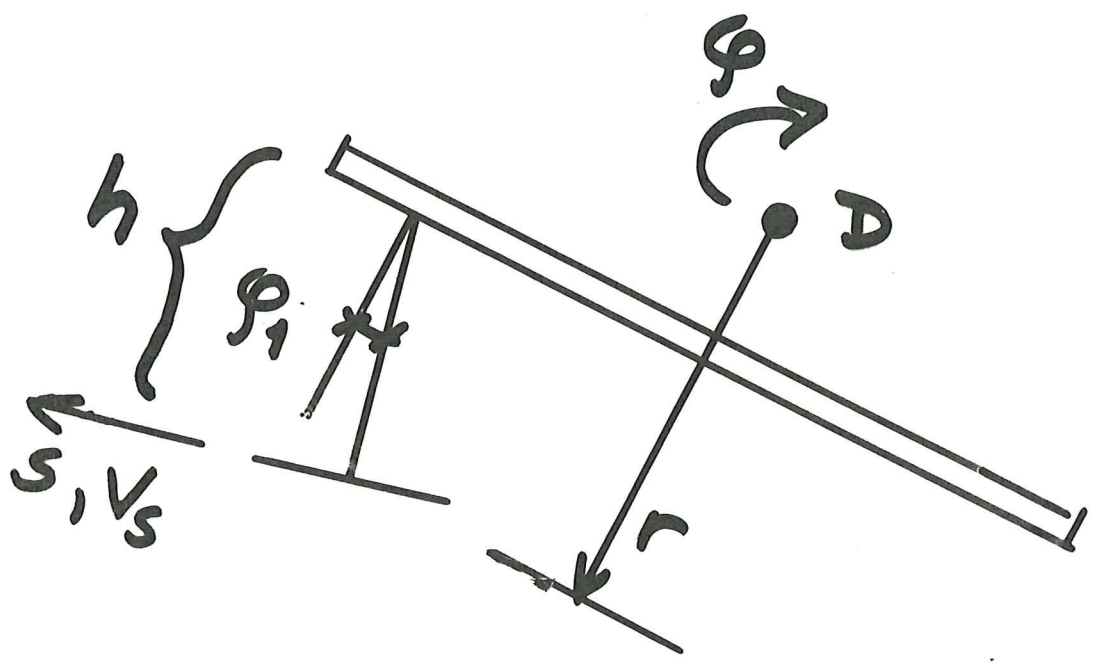
$$\omega = \int_s r ds$$

Gir

$$u(x, s) = u_0(x) - \varphi_{,x} \cdot \omega - v_{D,x} \cdot y - w_{D,x} \cdot z$$

$$\varphi_{,x} \equiv \frac{d\varphi}{dx} \equiv \frac{\partial \varphi}{\partial x}$$

Underflens :

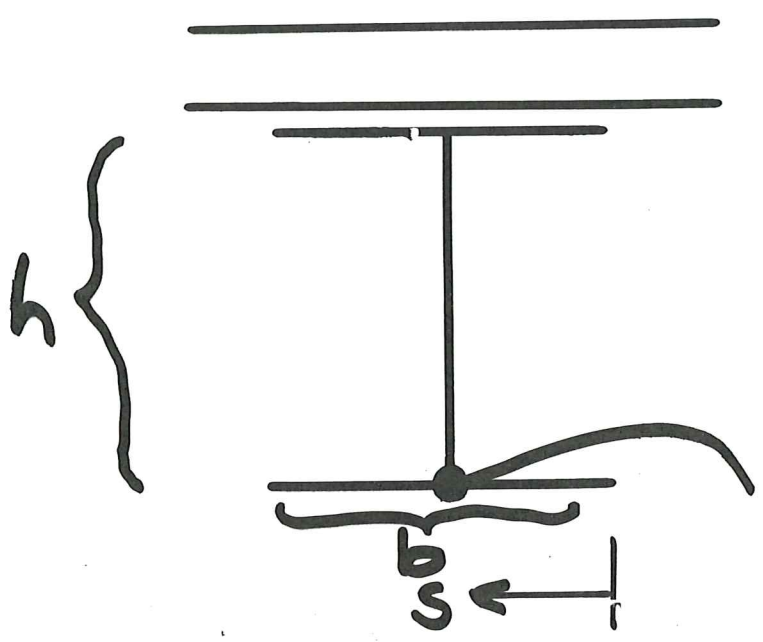


$$v_s = \varphi \cdot r - \underline{\varphi_1 \cdot h} + v_D \cdot \frac{\partial y}{\partial s} + w_D \cdot \frac{\partial z}{\partial s}$$

Gir

$$u(x,s) = u_0(x) - \varphi_{1,x} \cdot \omega + \underline{\varphi_{1,x} \cdot \bar{\omega}} - v_{D,x} \cdot y - w_{D,x} \cdot z$$

$$\bar{\omega} = \int h ds = h \cdot s + C$$

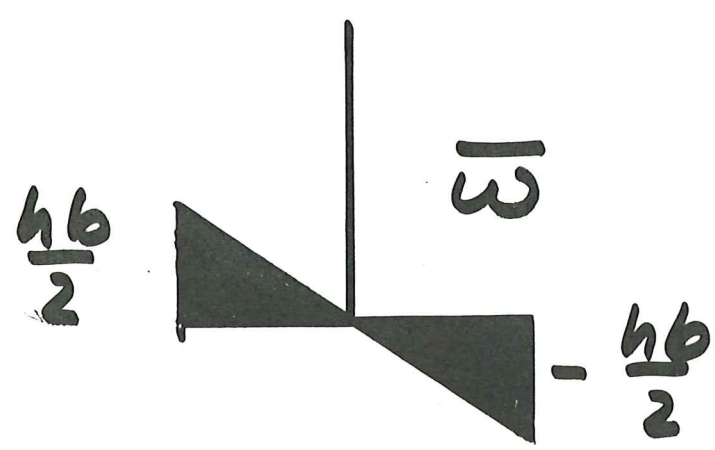


$$\bar{\omega} = h \cdot s + C$$

kontinuitet i u gir $\bar{\omega} = 0$ her

Dvs.

$$h \cdot \frac{b}{2} + C = 0 \quad \therefore \underline{C = -\frac{hb}{2}}$$



Rent momentbidrag i underflens.

Virtuelt arbeid:

$$\int_V \sigma_x \cdot \delta \epsilon_x dV + \int_V \tau \cdot \delta \gamma dV$$

Hvelvning
 Aksialdef.
 Bøyning

 St. Venant

$$= \int_{S_0} (t_x \cdot \delta u + t_y \cdot \delta v + t_z \cdot \delta w) dS$$

Ytre laster

Må finne uttrykk for
 ϵ_x og σ_x .

Toyning:

$$E_x = U_{,x} =$$

$$U_{0,x} - \rho_{,xx} \omega + \underbrace{\rho_{1,xx} \bar{\omega}}_{\text{For underftens}}$$

$$- V_{D,xx} y - W_{D,xx} z$$

Virtuell:

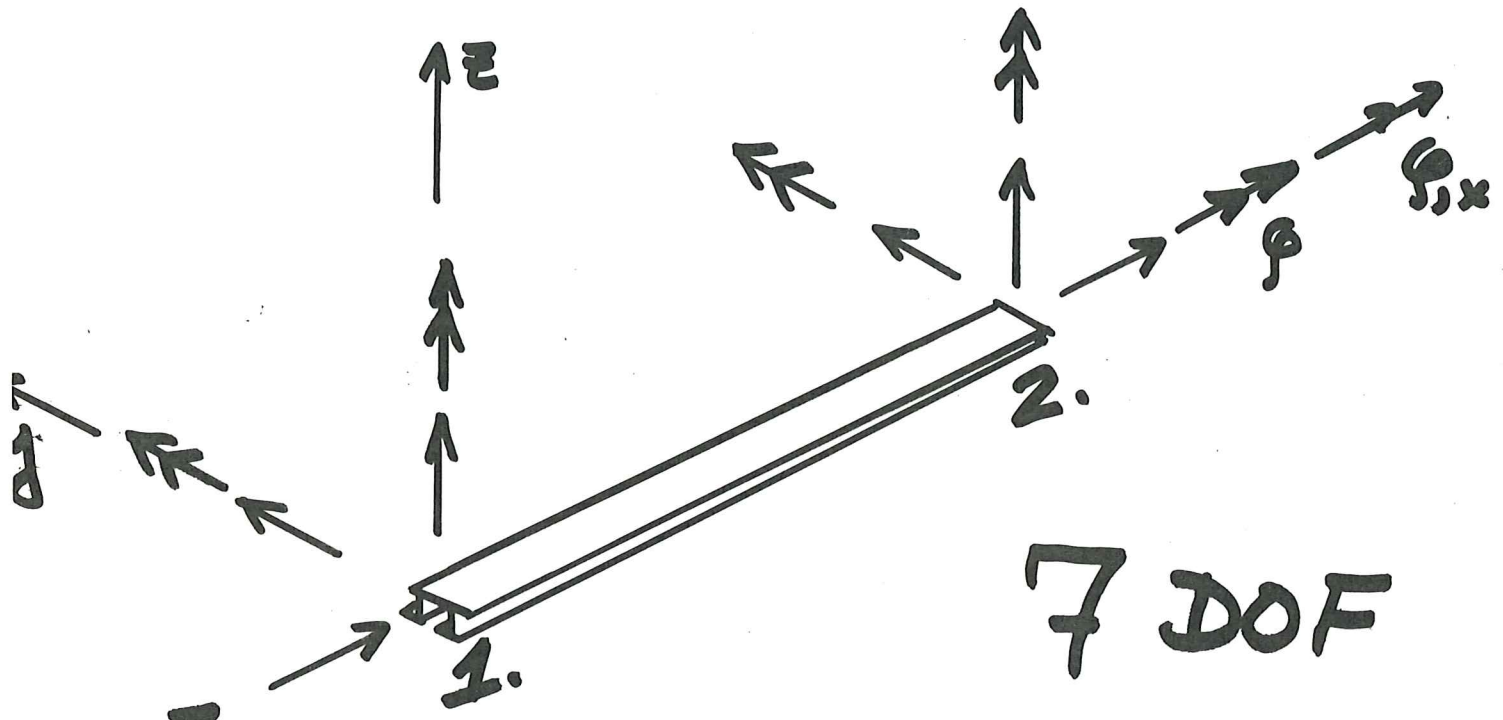
$$\delta E_x = \delta U_{0,x} - \omega \cdot \delta \rho_{,xx} - \bar{\omega} \cdot \delta \rho_{1,xx} \\ - y \cdot \delta V_{D,xx} - z \cdot \delta W_{D,xx}$$

Spanning:

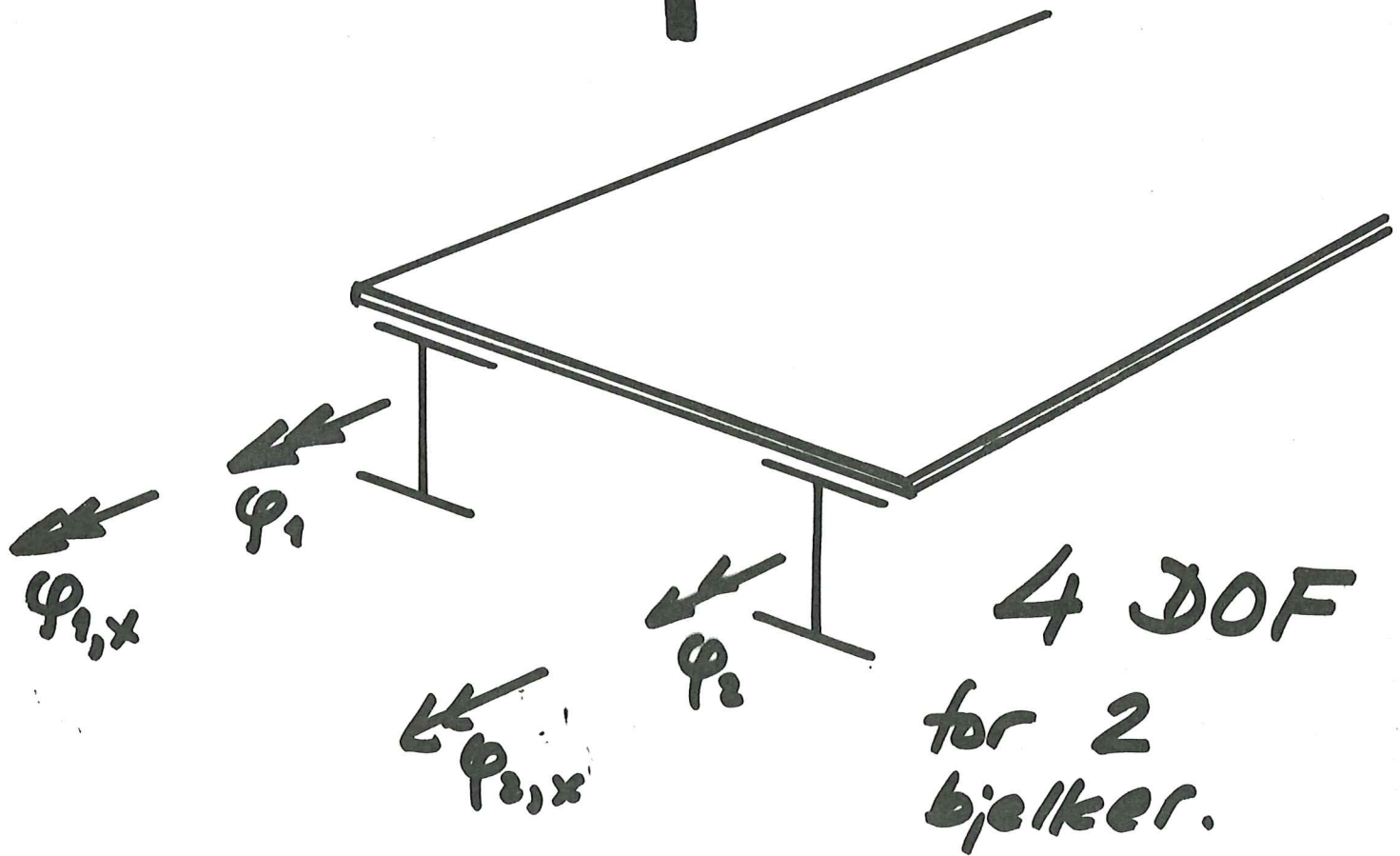
$$\sigma_x = E \cdot \epsilon_x$$

$$= E \left(u_{0,x} - \omega \rho_{1,xx} + \bar{\omega} \rho_{1,xx} - y \cdot v_{D,xx} - z \cdot w_{D,xx} \right)$$

ELEMENTMETODEN



+



Interpolasjon:

$$\varphi(x) = N_{\varphi}^T \varphi = N_{\varphi}^T \begin{Bmatrix} (\varphi) \\ (\varphi_{1,x})_1 \\ \vdots \\ (\varphi) \\ (\varphi_{1,x})_2 \end{Bmatrix}$$

$$\varphi_1(x) = N_{\varphi_1}^T \varphi_1 = N_{\varphi_1}^T \begin{Bmatrix} (\varphi_1) \\ (\varphi_{1,x})_1 \\ \vdots \\ (\varphi_1) \\ (\varphi_{1,x})_2 \end{Bmatrix}$$

$$u(x) = N_u^T u = N_u^T \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$v(x) = N_v^T v = N_v^T \begin{Bmatrix} (v) \\ (\vartheta_2)_1 \\ \vdots \\ (v) \\ (\vartheta_2)_2 \end{Bmatrix}$$

$$w(x) = N_w^T w = N_w^T \begin{Bmatrix} (w) \\ (\vartheta_y)_1 \\ \vdots \\ (w) \\ (\vartheta_y)_2 \end{Bmatrix}$$

Virtuelt arbeid fra hvelvning + aksial + bøyning

$$\int_V \sigma_x \cdot \delta \epsilon_x dV =$$

$$\underbrace{EA \int_L u_{,x} \cdot \delta u_{,x} dx}_{\text{Aksial}} + \underbrace{EI_z \int_L v_{,xx} \cdot \delta v_{,xx} dx}_{\text{Bøyn. om z}}$$

$$+ EI_y \int_L w_{,xx} \cdot \delta w_{,xx} dx$$

Bøyn. om y

$$+ EI_w \int_L \varphi_{,xx} \cdot \delta \varphi_{,xx} dx$$

Kopp
Hvelvning
når tussnitt
beholder form



$$+ EI_{\bar{w}} \int_L \varphi_{1,xx} \cdot \delta \varphi_{1,xx} dx$$

K_{φ, φ}
Hvelvning-
stivhet
koplet til

→ Forts.

φ,

$$\left. \begin{aligned}
 & - EI \bar{w} \int_L \varphi_{,xx} \cdot \delta \varphi_{,xx} dx \\
 & \boxed{\text{Kobling } \varphi \leftrightarrow \varphi_1} \\
 & - EI \bar{w} \int_L \varphi_{1,xx} \cdot \delta \varphi_{,xx} dx
 \end{aligned} \right\} \begin{array}{l} \text{Sym.} \\ \textcircled{1} \end{array}$$

$$\left. \begin{aligned}
 & - EI \bar{w}_y \int_L v_{,xx} \cdot \delta \varphi_{,xx} dx \\
 & \boxed{\text{Kobling } \varphi_1 \leftrightarrow v} \\
 & - EI \bar{w}_y \int_L \varphi_{1,xx} \cdot \delta v_{,xx} dx
 \end{aligned} \right\} \begin{array}{l} \text{Sym.} \\ \textcircled{2} \end{array}$$

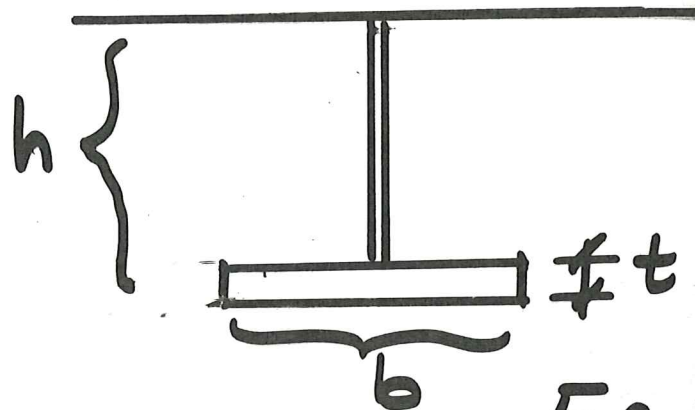
①: Reduksjon av torsjonsstivh.
 fra 

②: Reduksjon av y -stivhet
 fra samme.

Tverrsnittskonstanter:

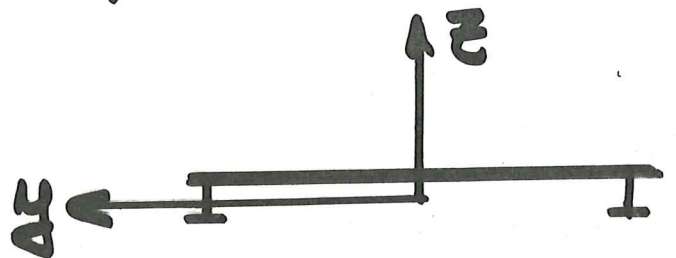
$$I_z = \int_A y^2 dA \quad ; \quad I_y = \int_A z^2 dA$$

$$I_w = \int_A w^2 dA$$



$$I_{\bar{w}} = \int_{A_{\text{Flens}}} \bar{w}^2 dA = \frac{h^3 b^3 t}{12} \quad \text{For hver u. flens}$$

$$I_{w\bar{w}} = \int_{A_{\text{Flens}}} w \cdot \bar{w} dA$$



$$I_{\bar{w}y} = \int_{A_{\text{Flens}}} w \cdot y dA \approx 0 \quad \text{stor bredde}$$

Sub - stivhetsmatriser:

Hvrlning + aksial + boyning

$$K_{uu} = EA \int_L N_{u,x} N_{u,x}^T dx \quad (2.2)$$

$$K_{vv} = EI_z \int_L N_{v,xx} N_{v,xx}^T dx \quad (4.4)$$

$$K_{ww} = EI_y \int_L N_{w,xx} N_{w,xx}^T dx \quad (4.4)$$

$$K_{\varphi\varphi} = EI_w \int_L N_{\varphi,xx} N_{\varphi,xx}^T dx \quad (4.4)$$

$$K_{\varphi,\varphi} = EI_w \int_L N_{\varphi,xx} N_{\varphi,xx} dx \quad (4.4)$$

Koblings-matriser:

Uveluning + bøyning

$$K_{\varphi\varphi} = \Theta EI \bar{w} \int_L N_{\varphi,xx} N_{\varphi,xx}^T dx$$

Symmetri

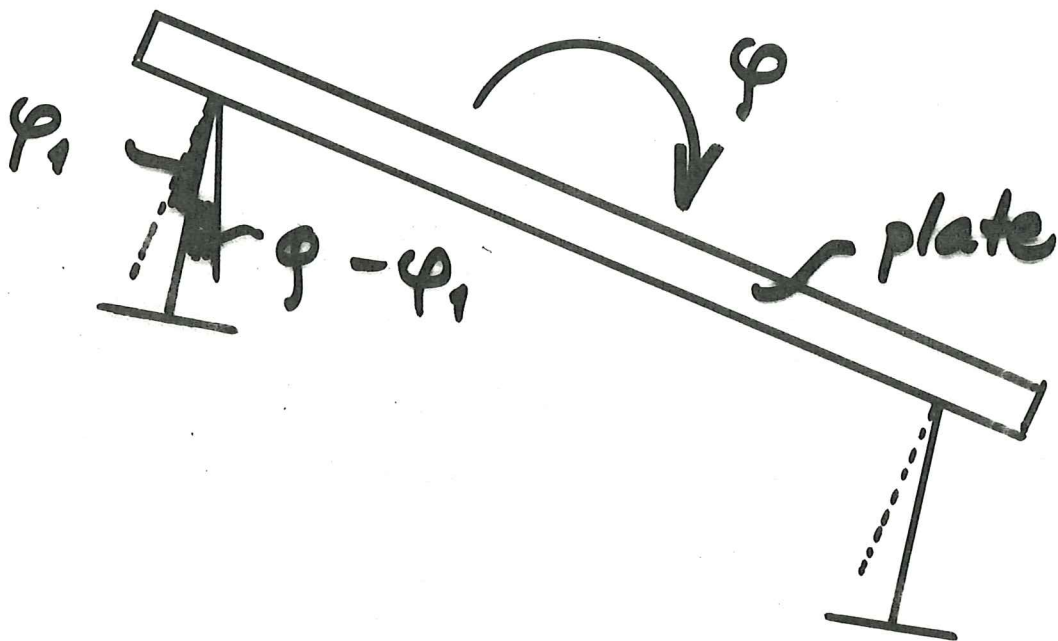
$$K_{\varphi, \varphi} = \Theta EI \bar{w} \int_L N_{\varphi,xx} N_{\varphi,xx}^T dx$$

$$K_{v\varphi} = \Theta EI \bar{w}_y \int_L N_{v,xx} N_{\varphi,xx}^T dx$$

Symmetri

$$K_{\varphi, v} = \Theta EI \bar{w}_y \int_L N_{\varphi,xx} N_{v,xx}^T dx$$

Virtuelt arbeid fra St. Venant



$$\int_V \vec{c} \cdot \delta \vec{\sigma} dV \rightarrow \int M_x \cdot \delta \varphi_{,x} dx$$

$$= \int_A G I_t \int_L \varphi_{,x} \cdot \delta \varphi_{,x} dx dA$$

$$= (G I_t)_{\text{plate}} \cdot \int_L \varphi_{,x} \cdot \delta \varphi_{,x} dx$$

$$+ \sum (G I_t)_I \cdot \int_L (\varphi_{,x} - \varphi_{1,x}) \cdot (\delta \varphi_{,x} - \delta \varphi_{1,x}) dx$$

Stivhetsmatriser fra St. Venant

Diagonal submatriser:

$$K_{\varphi\varphi} = (GI_t)_{\text{helt}} \int_L N_{\varphi,xx} N_{\varphi,xx}^T dx$$

$$K_{\varphi\varphi_1} = \sum_I (GI_t)_I \int_L N_{\varphi_1,xx} N_{\varphi_1,xx}^T dx$$

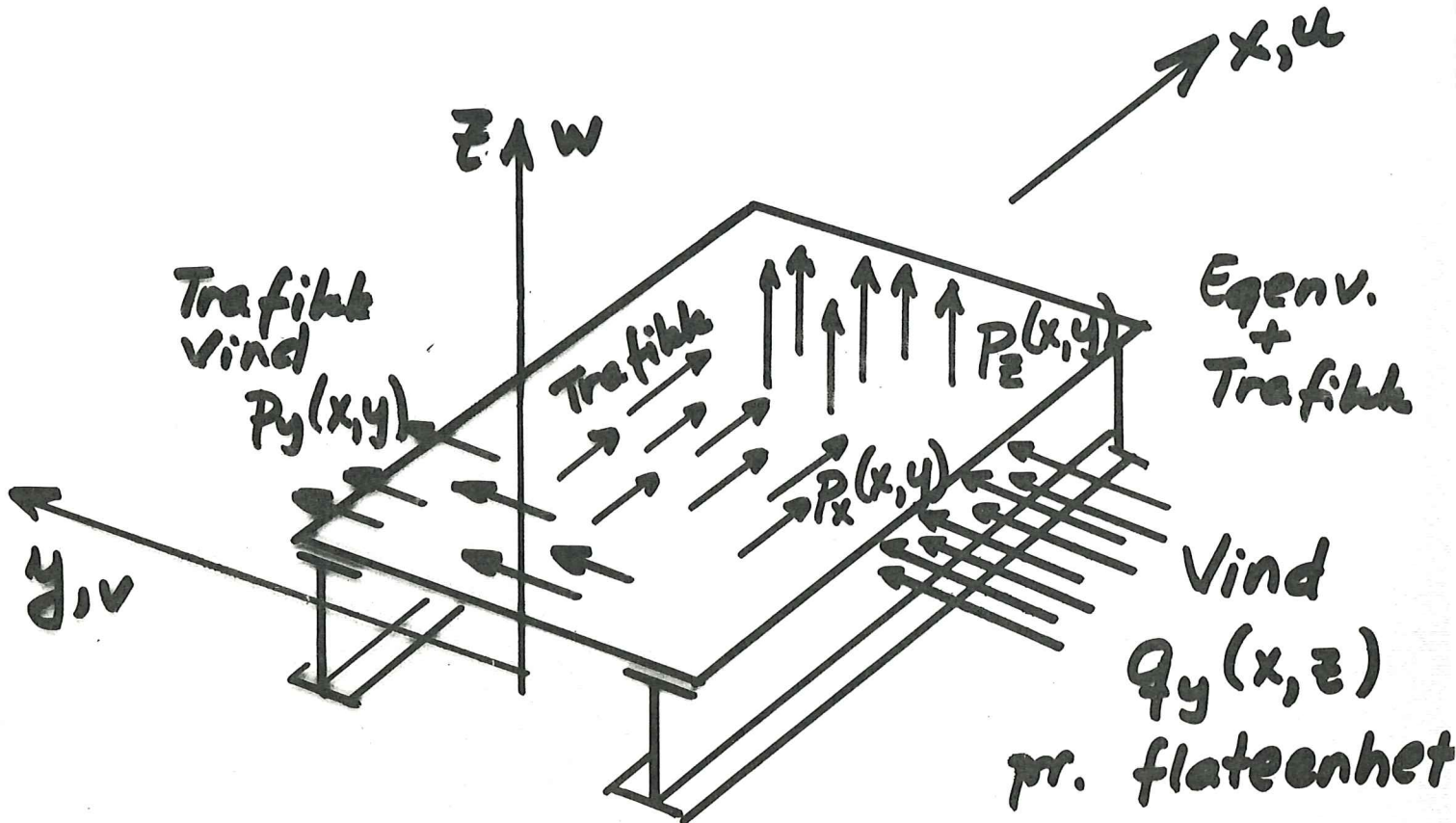
Kryss submatriser:

$$K_{\varphi\varphi_1} = \ominus \sum_I (GI_t)_I \int_L N_{\varphi,xx} N_{\varphi_1,xx}^T dx$$

Symmetri

$$K_{\varphi_1\varphi} = \ominus \sum_I (GI_t)_I \int_L N_{\varphi_1,xx} N_{\varphi,xx}^T dx$$

Ytre Laster



Last pr. arealenhet

Virtuelt arbeid fra ytre laster:

$$\int_{S_0} (p_x \delta u + p_y \delta v + p_z \delta w + q_y \delta v) dS$$

Aksiallast P_x

$$u(x, y) = u_0(x) - \varphi_{,x} \cdot \omega$$

$$- v_{D,x} \cdot y_p - w_{D,x} \cdot z_p$$

$$u(x, y) = N_u^T(x) u - \omega N_{\varphi,x}^T \varphi$$

$$- y_p N_{v,x}^T v - z_p N_{w,x}^T w$$

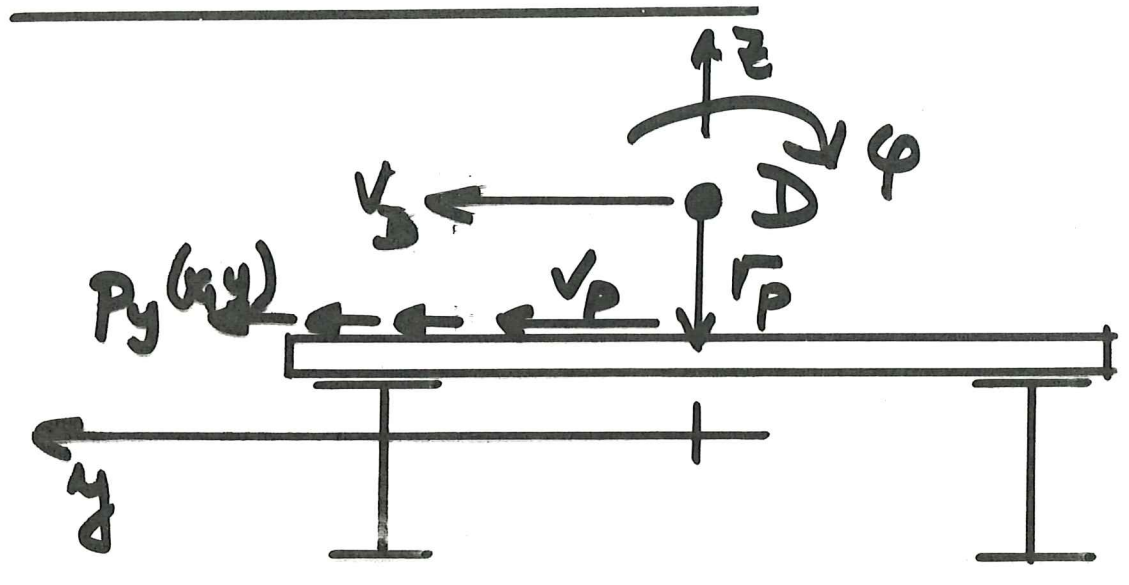
$$\delta u(x, y) = N_u^T \delta u - \omega N_{\varphi,x}^T \delta \varphi$$

$$- y_p N_{v,x}^T \delta v - z_p N_{w,x}^T \delta w$$

$$\begin{aligned}
 \int_{A_{pl}} p_x \delta u \, dA &= \int \delta u^T p_x \, dA \\
 &= \delta u^T \underbrace{\iint_{A_{pl}} N_u(x) \cdot p_x(x,y) \, dA}_{\mathcal{S}_u} \\
 &- \delta v^T \underbrace{\iint_{A_{pl}} y_p \cdot N_{v,x} \cdot p_x(x,y) \, dA}_{\mathcal{S}_v} \\
 &- \delta w^T \underbrace{\iint_{A_{pl}} z_p \cdot N_{w,x} \cdot p_x(x,y) \, dA}_{\mathcal{S}_w} \\
 &- \delta \varphi^T \underbrace{\iint_{A_{pl}} \omega \cdot N_{\varphi,x} \cdot p_x(x,y) \, dA}_{\mathcal{S}_\varphi}
 \end{aligned}$$

Også på
plass for bimom.
div. $\varphi_{,x}$ - plass.

Laterallast P_y



$$v_P = v_D + \varphi \cdot r_P$$

$$= N_v^T v + r_P N_\varphi^T \varphi$$

$$\delta v_P = N_v^T \delta v + r_P N_\varphi^T \delta \varphi$$

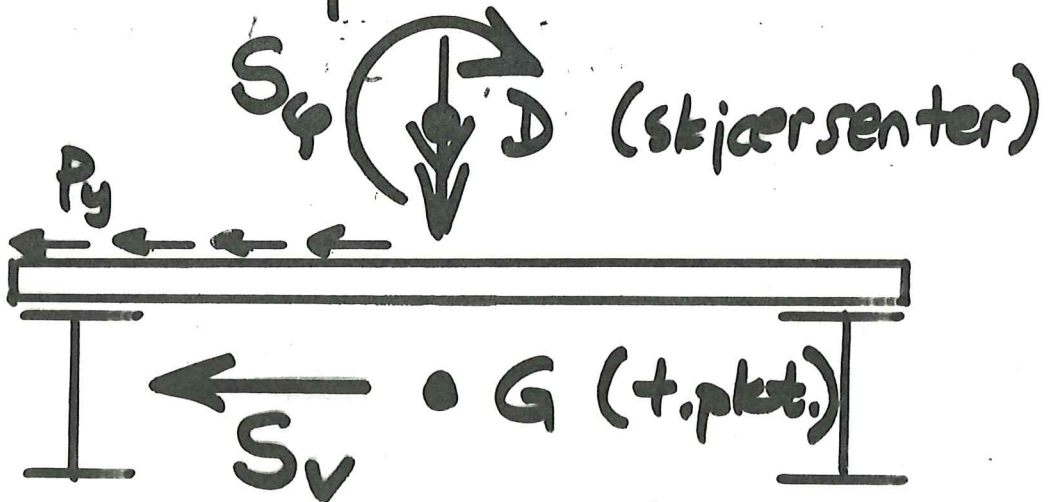
$$= \delta v^T N_v + \delta \varphi^T r_P N_\varphi$$

$$\int_{A_{pl}} p_y \delta v_p dA$$

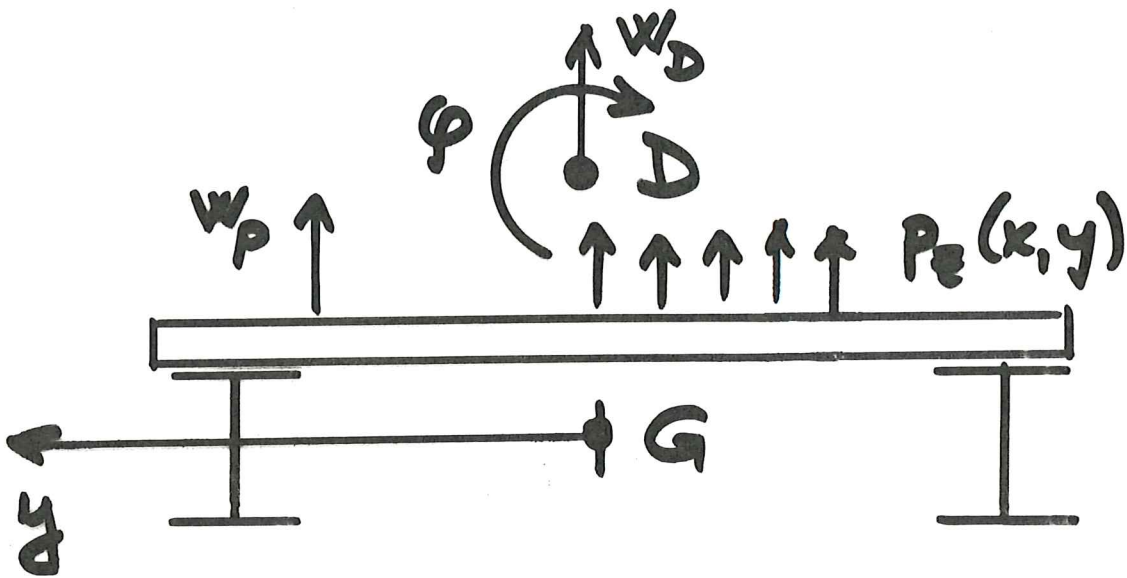
$$= \delta v^T \iint_{A_{pl}} N_v(x) \cdot p_y(x,y) dA \quad S_v$$

$$+ \delta \phi^T \iint_{A_{pl}} \overset{\epsilon_D - \epsilon_P}{r_p} N_\phi(x) \cdot p_y(x,y) dA \quad S_\phi$$

P_a plass for torsj. mom.
og binmoment



Vertikallast P_E



$$w_p = w_D + \varphi \cdot y$$

$$= N_w^T w + y \cdot N_\varphi^T \varphi$$

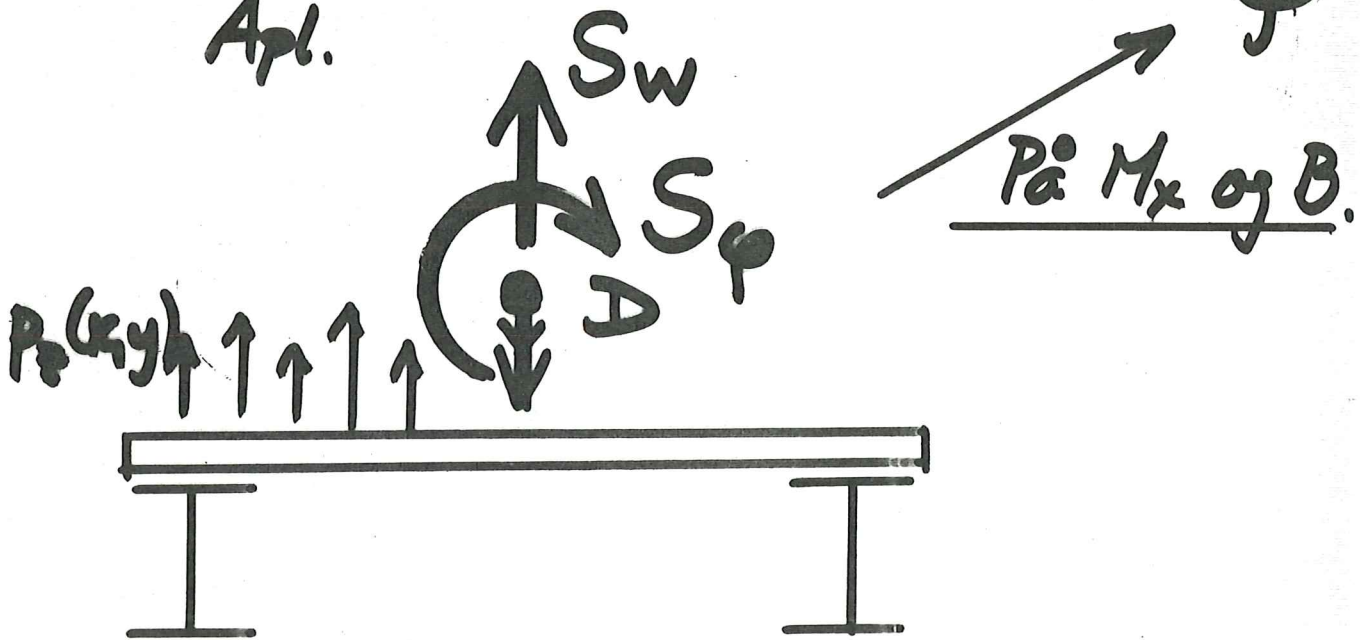
$$\delta w_p = N_w^T \delta w + y N_\varphi^T \delta \varphi$$

$$= \delta w^T N_w + y \delta \varphi^T N_\varphi$$

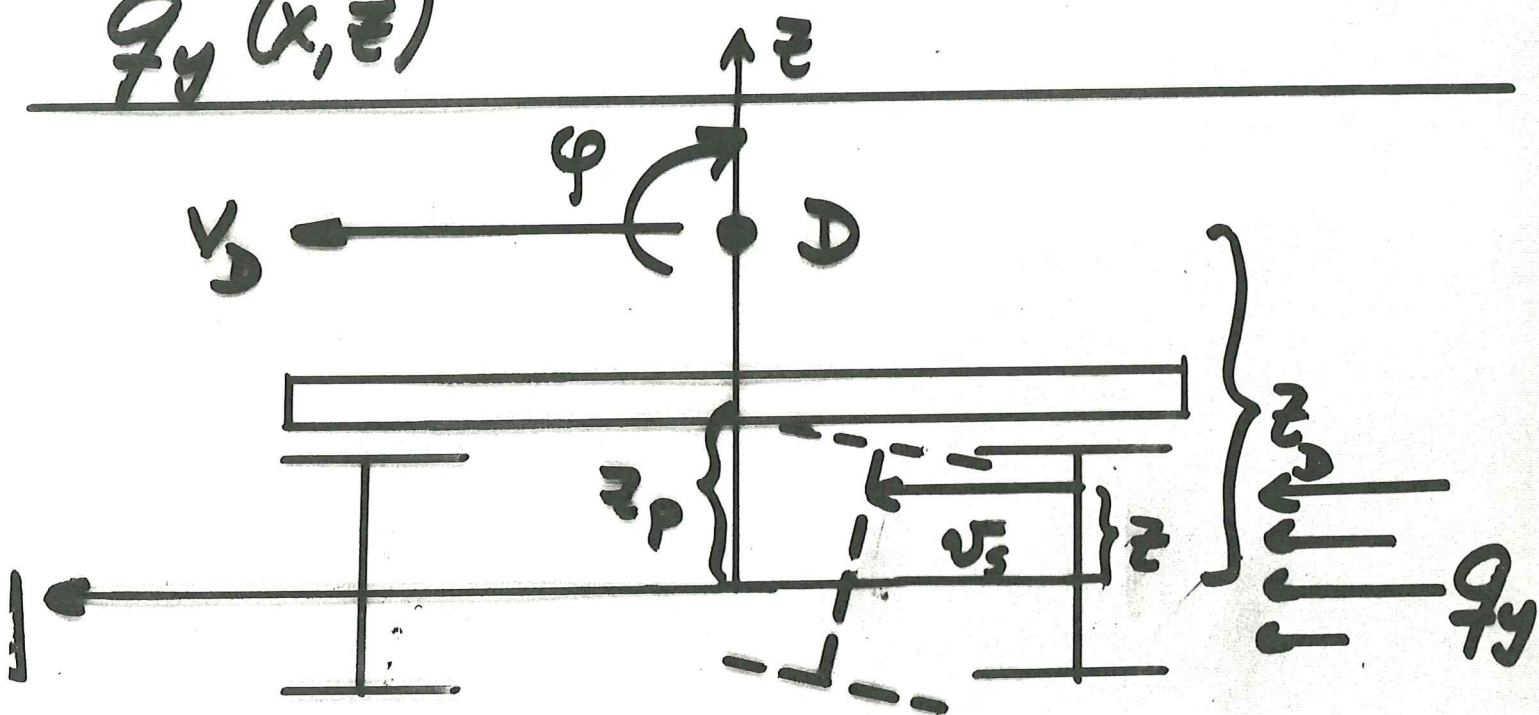
$$\int_{A_{plate}} p_z \cdot \delta w \, dA =$$

$$\delta w^T \underbrace{\iint_{A_{pl}} N_w(x) \cdot p_z(x,y) \, dA}_{S_w} \quad S_w$$

$$+ \delta \varphi^T \underbrace{\iint_{A_{pl.}} y N_\varphi(x) \cdot p_z(x,y) \, dA}_{S_\varphi} \quad S_\varphi$$



Vindlast på bærer

 $q_y(x, z)$


$$v_s(z) = v_D + \varphi \cdot (z_D - z) - \varphi_1 \cdot (z_P - z)$$

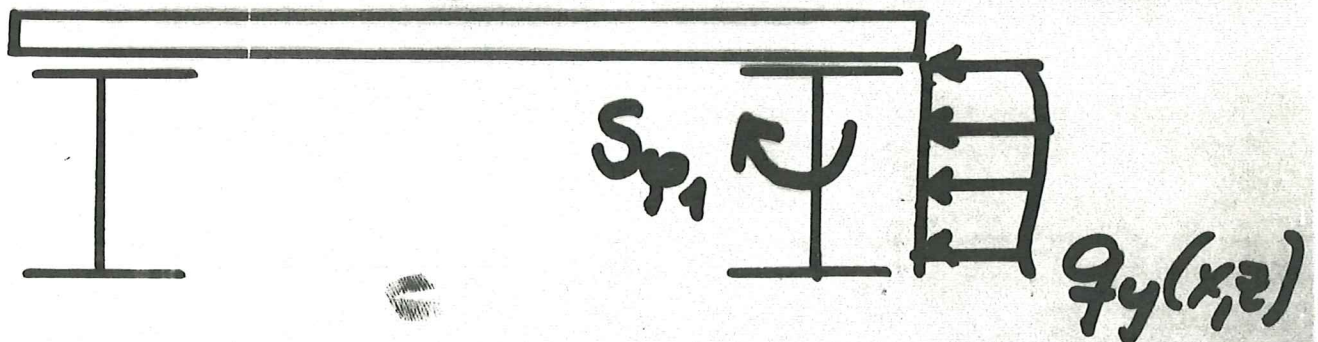
$$= N_v^T(x) v + (z_D - z) N_{\varphi}^T \varphi - (z_P - z) N_{\varphi_1}^T \varphi_1$$

$$\int_{A_{\text{step}}} q_y \cdot \delta v \, dA =$$

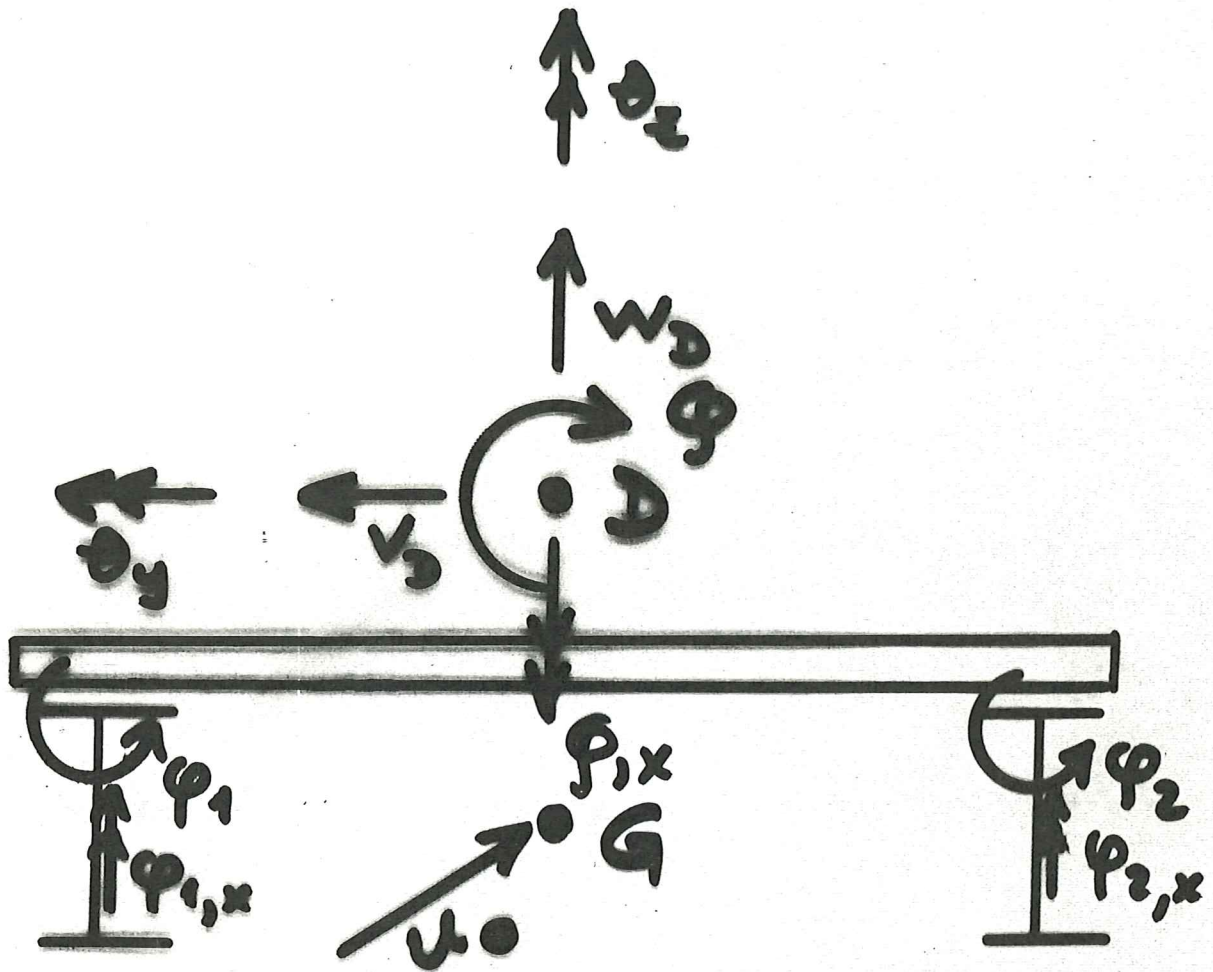
$$\delta v^T \int_{x_{\text{step}}} \int_{\xi} N_v(x) \cdot q_y(x, z) \, dz \, dx \quad S_v$$

$$+ \delta \varphi^T \int_{\text{step}} \int N_\varphi(x) \cdot (z_0 - z) \cdot q_y(x, z) \, dz \, dx \quad S_\varphi$$

$$\div \delta \varphi_1^T \int_{\text{step}} \int N_{\varphi_1}(x) \cdot (z_p - z) \cdot q_y(x, z) \, dz \, dx \quad S_{\varphi_1}$$



Parameter



11 DOF pr. node

22 DOF pr. element

Temperaturlast:

$$\sigma_x = E(\epsilon_x - \alpha T)$$

$$\epsilon_x = u_{0,x} - \omega \varphi_{,xx} + \underbrace{\bar{\omega} \varphi_{,xx}}_{\text{Flens}} - y \cdot v_{D,xx} - z \cdot w_{D,xx}$$

som for

Virtuell:

$$\begin{aligned} \delta \epsilon_x = & \delta u^T N_{u,x} - \delta \varphi^T N_{\varphi,xx} \cdot \omega \\ & + \delta \varphi^T N_{\varphi,xx} \cdot \bar{\omega} \\ & - \delta v^T N_{v,xx} \cdot y - \delta w^T N_{w,xx} \cdot z \end{aligned}$$

$$\int \sigma_x \cdot \delta \epsilon_x \, dV$$

gir som tidligere stivheter
+ fra temperatur:

$$- \delta u^T \int_L \int_A N_{u,x} \cdot E \alpha T \, dA \, dx$$

$$+ \delta \varphi^T \int_L \int_A N_{\varphi,x} \cdot \omega E \alpha T \, dA \, dx$$

$$- \delta \varphi_1^T \int_L \int_A N_{\varphi_1,x} \cdot \bar{\omega} E \alpha T \, dA \, dx$$

Flens

$$+ \delta v^T \int_L \int_A N_{v,xx} \cdot y \cdot E \alpha T \, dA \, dx$$

$$+ \delta w^T \int_L \int_A N_{w,xx} \cdot z \cdot E \alpha T \, dA \, dx$$

Temperatur - laster :

$$S_u = \int_L \int_A N_{u,xx} \cdot E \alpha T dA dx$$

$$S_v = - \int_L \int_A N_{v,xx} \cdot y \cdot E \alpha T dA dx$$

$$S_w = - \int_L \int_A N_{w,xx} \cdot z \cdot E \alpha T dA dx$$

$$S_\varphi = - \int_L \int_A N_{\varphi,xx} \cdot \omega \cdot E \alpha T dA dx$$

$$S_{\varphi_1} = \int_L \int_A N_{\varphi_1,xx} \cdot \bar{\omega} \cdot E \alpha T dA dx$$

Flens

Svinn i betong:

Analogt temperatur

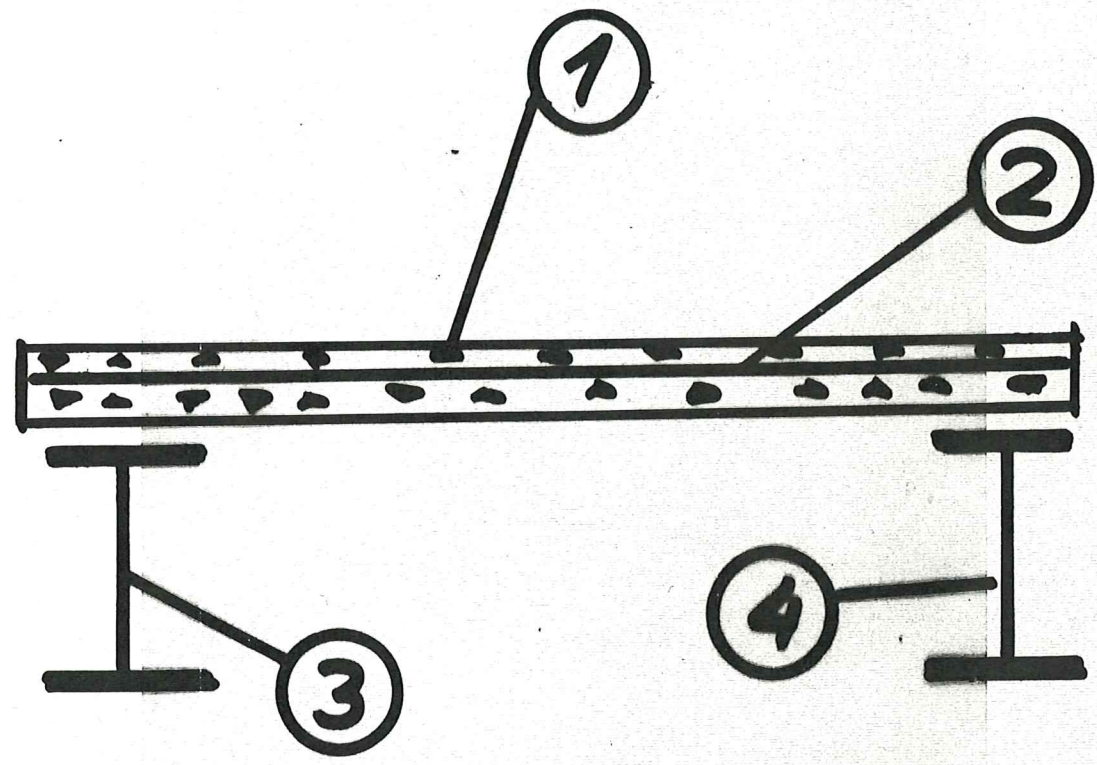
$$\alpha T \rightarrow \epsilon_s$$

Kryp i betong:

$$E = \frac{E_c}{1 + \varphi}$$

Benyttes for længtid's last.

Tverrsnittsdeler:



- ① Betong plate
- ② Langsgående arm.
- ③ Stålbjelke
- ④ Stålbjelke